ASTRONOMY AND SPACE SCIENCE
eds. M.K. Tsvetkov, L.G. Filipov, M.S. Dimitrijević, L.Č. Popović,
Heron Press Ltd, Sofia 2007

Large Scale Structures in the Universe. III. Cluster Analysis of Faint Galaxies in the Direction of HERCULES Void

G.T. Petrov[1], **J.W. Fried**[2], **A.Y. Kniazev**[3]

[1]Institute of Astronomy, Bulgarian Academy of Sciences,
72 Tsarigradsko chaussee, 1784 Sofia, Bulgaria

[2]Max Plank Institute for Astronomy, Königstuhl 17, D 69117 Heidelberg, Germany

[3]South African Astronomical Observatory, P.O. Box 9, Observatory, 7935, South Africa

Abstract. Based on photographic plates obtained with 2m RCC telescope at National Astronomical Observatory "Rozhen" (Bulgaria) we inspected an area of $\sim$1 sq.degree in the direction of the Hercules void with automatic object detection software. (Hercules void). The coordinates (2000), magnitudes m(B), diameters, position angles surface brightness and some morphological parameters for 1851 faint galaxies detected in a field centered at 1600+18 (1950) in the wide range of magnitudes $13^m \leq \mathrm{B} \leq 21^m$ and effective surface brightnesses $16 \leq \mu_{\mathrm{eff}}(\mathrm{B}) \leq 24$ mag arcsec^{-2} have been determined and studied using cluster analysis technique. K_means method were used to determine the substructures in the distribution of faint galaxies. The groups of Low surface brightness galaxies (Primeval(?) galaxies) as the ones with High Surface Brightness were detected in such manner. Edge_on galaxies are almost not selected because of bias effects of the discrimination between stars and galaxies. Cluster analysis of the distribution of Concentration index (a parameter outlined morphology of the object) give us some possibility for semiautomatic separating or classifying of galaxies as spiral and elliptical ones. Method to select Active galactic nuclei amogst faint galaxies is demonstrated.

1 Introduction

The presence of voids in the distribution of galaxies has been discovered in early redshift surveys of galaxies - see e.g. Chincarini & Rood [1], Gregory & Thompson [2]. Further studies show that the largest voids are those delineated by rich clusters and superclusters of galaxies – Oort [3], Rood [4]. The Hercules region has attracted the attention of astronomers since Shapley [5] discovered the Hercules supercluster covers a large area of the sky north of celestial equator in the range ascension range between $12^h < \alpha < 18^h$. In this third part of our study we present results from cluster analysis of different parameters of faint galaxies in the direction of Hercules void - magnitudes, surface brightness, diameters, position angles, concentration indices etc. For part 1 and 2 see Petrov et al. [6], [7].

In astronomy cluster analysis technique is used in wide field of investigation – from classification and star-galaxy separation to the photometrically defined spectral classes, taxonomy construction and distribution of asteroids, stars and galaxies. Some examples and bibliography could be find in Murtagh & Heck [8] and Murtagh [9]. Multivariate statistics recently are reviewed in an astronomical context by Feigelson&Babu [10]. Many monographs and WEB based courses presenting multivariate statistics are available, such as Fung [11], Franti([12]. Cluster *k*-Means is used in this analysis as it is suitable for very large data sets and handles mixed data types that can contain missing values, contiguity constraints, magnitudes, etc.

Detailed explanation of the observatioal material and basik reduction steps could be found in Kniazev et al. [14] and Petrov et al. [7].

We selected galaxies with angular sizes $D \geq 5$ arcsec, where D is the major-axis diameter at the limiting isophote of the GMPlus scans of our plates ($\mu_{\rm lim}(B) = 26.5 \pm 0.5$ mag arcsec^{-2})≡ SBB_90.

Throughout the paper a Hubble constant H_0 = 75 km s^{-1} Mpc^{-1} is adopted.

2 First Astrometry and Photometry Results

The data for all 1851 galaxies in the field – coordinates, aperture and surface photometry, position angles, diameters, axis ratio and concentration are presented as FITS table **v1600gal.fts** and are available under question.

Galaxy Number Counts: The number counts of galaxies as a function of magnitude is one of the classical cosmological tests. We did it for our data and the result is plotted on the right panel of Figure 1. Galaxy number counts are shown in 0.5 mag bins. The errors bars correspond to Poisson noise. The line in Figure 1 shows a fit to the galaxy counts-magnitude relation expected in a homogeneous universe assuming Euclidean geometry for three-dimensional space. The observed galaxy counts are quite consistent with this line for $17^m \leq B \leq 20^m$ and even fainter up to B=20.5 mag. It means that we have complete data up to this magnitudes. With our data we found big excess of bright galaxies ($B < 17.0^m$).

After the Goodness-of-Fit Tests for Normal Distribution of B_msk we can reject the idea that B_msk comes from a Normal ones with 95% confidence. Probability plot distribution gave us indirect proofs *the distribution comes at least from two independant selection*. In our opinion brighter galaxies could be nearer to us and faint end of the distributin will come from the direction of the void itself. Superposition of these subselection could explain the excess we marked here.

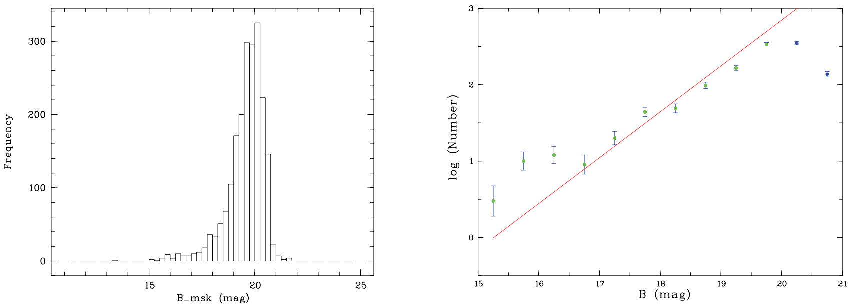

Figure 1. Left panel: Distribution of apparent magnitudes for all galaxies detected with our programs. Right panel: Number counts of all galaxies detected with our programs as a function of apparent magnitude. The errors bars on the galaxy counts are Poissonian. The line shows the count-magnitude relation expected for a homogeneous galaxy distribution in a universe with "Euclidean" geometry: $N(B) = A_B \cdot 10^{0.6B}$.

Index of Concentration: Our programs calculate a number of global parameters for every galaxy. Some of these may be useful for morphological galaxy classifications (Strateva et al. [15]). A particularly useful parameter is the concentration index, defined as the ratio of the radii containing 90% and 50% of a galaxy's light. For the classical de Vaucouleurs profile (E-S0 galaxies) C is ~5.5 and for pure exponential disks (S galaxies) C~2.3. These values are valid for the idealized seeing-free case.

3 Clustering in the Distribution of Faint Galaxies

THE HERCULES SUPERCLUSTER (Moles et al., [16]) is made up of three Abell clusters A2151, A2147, and A2152, with a connection toward the A2197/A2199 supercluster (Chincarini et al. [17]). The structures of the Hercules supercluster have been analysed with the non-hierarchical descendant taxonomical method, using α, δ and vT as active coordinates, and morphological type, position angle and apparent magnitude as control parameters. Here we present cluster analysis of faint galaxies in the direction of the Hercules void. Based on the several tests we select 5 clusters as predefined number of cluster to determine.

Surface Brightness of Galaxies: Figure 2 represents the results from cluster analysis of Diameters, Position angles, Index of concentration, Surface brightness, Axis Ratios and Integrated magnitudes distributions. Surface brightness of galaxies after correction for the absorption in the Galaxy (see Arakelian, [18] for details) is determined by the equation:

$$B = m_p - 0.25\text{cosec}|bII| + 2.5 \log \pi \frac{ab}{4} + 0.22(a/b) + 0.73. \qquad (1)$$

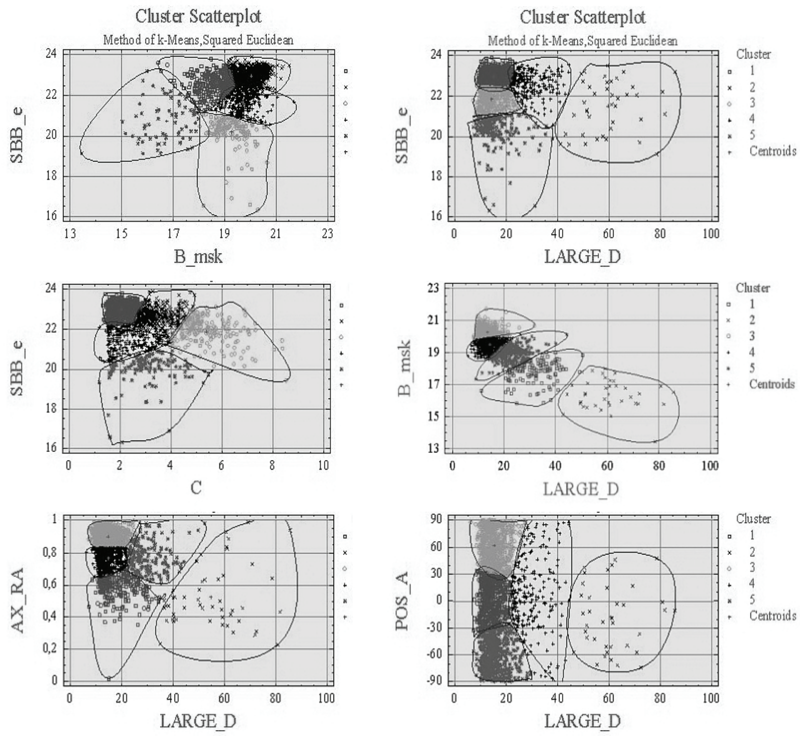

Figure 2. Cluster analysis of: Surface Brightness SBB_eff against Total magnitude B_msk, Index of Concentration C and diameters Large_D and of Large diameters Large_D against Total magnitude B_msk, Axis ratio AX_RA and position angle Pos_A.

These surface brightness are close to the Holmberg system (Holmberg [19]). As in Arakelian [18] High Surface Brightness Galaxies (HSBG) are the galaxies with SB $<=$ 22.0 mag/sqr.arcsec. Such selection split all the objects on the field in two groups - 463 galaxies with HSB and 1388 LSB galaxies (LSBG).

Although the separation of the groups defined in such manner is not so clear, relations C - SBB_eff and B_msk - SBB_eff mark such separation. Centroids from the Tab. 1 (clusters numbers 1 and 3) defined ca. 1200–1300 LSBG.

As a result of cluster analysis of surface brightness, magnitudes, diameters, concentration index, position angles and axis ratio of the galaxies, one can split all the objects in two main groups – 463 HSBG – i.e. SBB_eff$<=$22.0 mag arcsec^{-2} and 1388 LSBG – i.e. SBB_eff$>$22.0 mag arcsec^{-2}.

Below the two groups will be studied in more details.

Table 1. V1600+18 – Cluster Analysis Summary – k_Means Centroids

Cl	Memb	Perc	B_msk	SBB_e	Memb	Perc	C	SBB_e
1	818	44,19	20,10	23,02	397	21,45	3,45	22,65
2	91	4,92	16,93	21,00	129	6,97	3,08	19,97
3	400	21,61	18,81	22,72	897	48,46	2,06	23,00
4	188	10,16	19,39	20,42	183	9,89	5,39	21,86
5	354	19,12	19,69	22,0	245	13,24	2,27	21,46
Cl	Memb	Perc	D	SBB_e	Memb	Perc	D	B_msk
1	381	20,58	14,20	21,86	741	40,03	13,69	20,24
2	40	2,16	59,57	21,50	38	2,05	60,29	16,42
3	1008	54,46	14,92	22,98	138	7,46	31,21	17,84
4	191	10,32	15,89	20,28	619	33,44	14,21	19,47
5	231	12,48	29,17	22,65	315	17,02	22,41	19,02
Cl	Memb	Perc	D	AX_RA	Memb	Perc	D	POS_A
1	603	32,58	14,56	0,76	556	30,04	15,31	+61,73
2	249	13,45	18,37	0,58	37	2,00	60,92	-18,16
3	169	9,13	30,08	0,77	212	11,45	30,16	+6,97
4	780	42,14	14,65	0,90	534	28,85	14,12	+0,63
5	50	2,70	54,75	0,52	512	27,66	15,43	-61,75

Low Surface Brightness Galaxies (LSBG): LSB galaxies are a subgroup of the general galaxy population and comprise primarily extended, quiescently evolving disk galaxies, irregular galaxies, and dwarf galaxies lacking extended starbursts. They are one of the main constituents of the realm of galaxies. They are usually defined as objects with a blue central surface brightness $\mu_0(B)$ significantly fainter than the Freeman value of 21.65 mag arcsec^{-2} (Freeman [20]). However, the threshold value of $\mu_0(B)$ to classify galaxies as LSB galaxies varies in the literature from $\mu_0(B) \geq$ 23.0 mag arcsec^{-2} (Impey & Bothun [21]) to $\mu_0(B) \geq$ 22.0 mag arcsec^{-2} (Impey et al. [23]). There are many topics for which the knowledge of the properties of the LSB galaxy population is crucial. They include the following: a) the galaxy luminosity function, especially at its faint end, see e.g. Dalcanton [25], Trentham & Tully [26], Cross & Driver [27] which in turn is related to the understanding of the primordial power spectrum of density fluctuations – e.g. Ostriker [28]; b) the spatial distribution of lower-mass galaxies, which allows us to check the predictions of cold dark matter cosmology for large scale structure formation – see e.g. Peebles [29]; c) the physics of star formation at low gas surface densities - (vanZee et al. [30], Ferguson et al. [31], Noguchi [32]; d) the role of interactions in galaxy evolution; and many others.

The detection of LSB galaxies is difficult due to their intrinsically low global luminosities and their characteristic low surface brightness. Despite more than 20 years of LSB galaxy studies, their census remains incomplete.

Past LSB surveys were usually either large area photographic surveys or deep CCD surveys with small area – for a nice review see Dalcanton et al. [33].

An interesting aspect of this study is the identification of a substantial number of probably *luminous distant galaxies* with Low surface Brightness. In Figure 3 from the relations between the large diameters, the effective surface brightnesses $\mu_{\rm eff}$ and the integrated magnitude some LSBGs could be selected there – bright galaxies with B$\sim$15.5–18 mag or such ones with large diameters ($>$ 40 arcsec). Amongst 1388 galaxies with SB $>$ 22 mag/sqr.sec, *a group of 76 LSB objects are selected* with B $<$ 19 mag and D $>$ 30 arcsecs.

Giant Low Surface Brightness Galaxies: As above bright galaxies (i.e. with lower magnitudes) with large diameters could be a good candidates for LSBG group. This again is very well demonstrated from the *k*_Means clustering on Figure 3 and centroids of the clusters. The method outlines a group of galaxies with magnitudes ca. 16.3 and diameters ca. 60 arcsec! These galaxies could fall into the category of the so-called *giant LSB galaxies*, or 'cousins' of Malin 1 (Bothun et al. [34]), but the distances to galaxies are needed to say this definitely. In Fig. 3 the relations between the effective surface brightnesses $\mu_{\rm eff}$ and large diameters are plotted and clustered. Some LSBGs could be selected there – bright galaxies with B$\sim$15.5–18 mag or such ones with large diameters ($>$ 40 arcsec) and SB $>$ 22 mag/sqr.sec. The properties of giant LSB galaxies are summarized in the paper by Sprayberry et al. [35]. Currently, only ca. 25 galaxies of this type are known. As Bothun et al. [36] emphasize, giant LSB galaxies are quite enigmatic from the point of view that they normally formed their spheroidal component, but no conspicuous stellar disk ever formed around their bulge. Improved statistics for these objects will lead to a better understanding of the relationship between their fundamental parameters and the conditions/processes that led to their formation. The high detection rate of giant LSB galaxy candidates with our programs promises substantial progress for future systematic studies.

High Surface Brightness Galaxies: From the eq. 1 faint galaxies with large diameters could be a good candidates for HSBG group. This is very well demonstrated from the *k*_Means clustering on Figure 3 and centroids of the clusters. The method outlines a well populated group of galaxies with magnitudes ca. 17.5 and diameters ca. 32–36 arcsec. An intersting results of this study is very well outlined group of very high surface brightness galaxies – these with SBB_eff between 16 and 19 mag/sqr.sec. This group in some variations of the members is visible in all distributions – from magnitudes to position angles. Alltogether *77 well defined HSB galaxies*, selected after analysis of the histograms of diameters and magnitudes of 463 HSBG with diameters, larger than 20 arcsecs and magnitudes, larger than 17.0.

In Figure 3 clustering for HSBG – right panel and LSBG – left panel are shown.

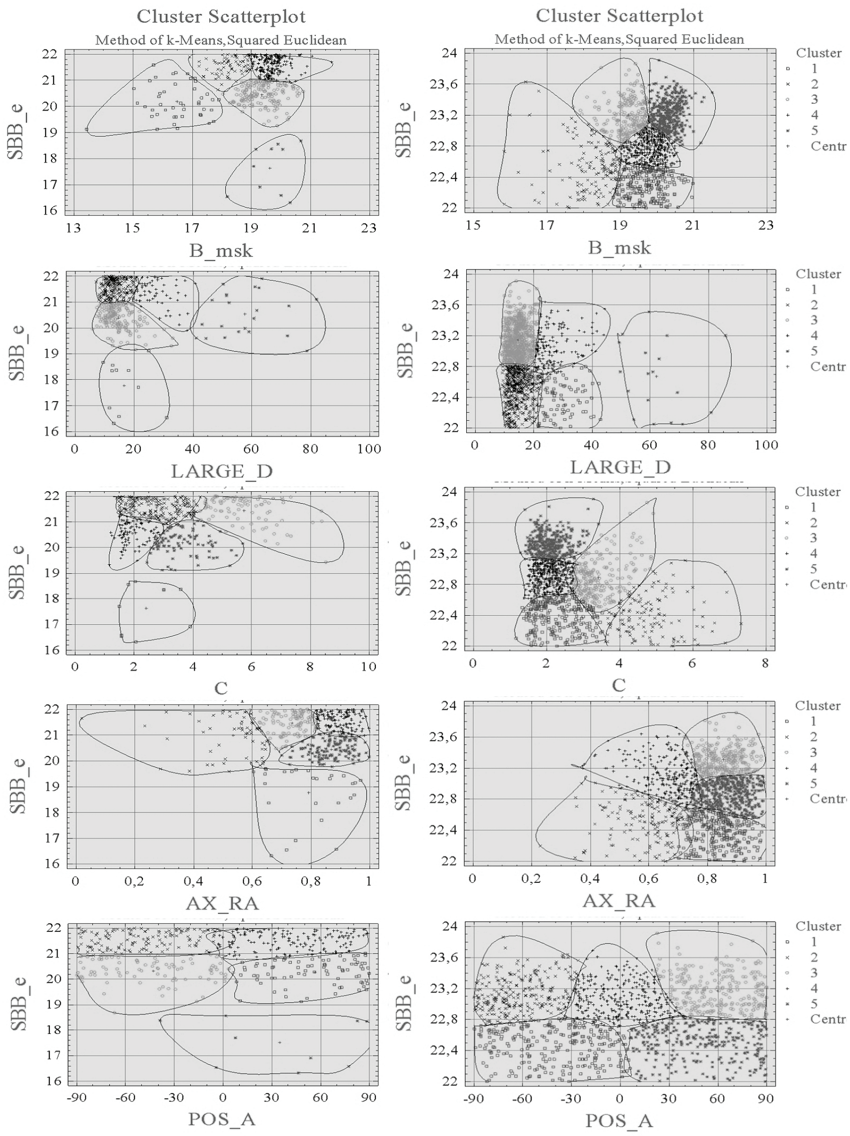

Figure 3. Clustering SBB_{eff}, B_msk, C and Large_D for: Right panel - HSBG, Left panel - LSBG.

Morphological Segregation of Galaxies: Our photometric programs calculate a number of global morphological parameters for every galaxy. Some of these may be useful for morphological galaxy classifications (see earlier discussions by, e.g. Kent [37], Kodaira et al. [38], Fioc & Rocca-Volmerange [39], Shimasaku et al. [40]. A particularly useful parameter is the concentration index (C hereafter), defined as the ratio of the radii containing 90% and 50% of a galaxyś light. As mentioned above for the idealized seeing-free case for the classical de Vaucouleurs profile – (Strateva et al. [15]), C is ∼5.5, and for pure exponential disks, C ∼2.3. In our case due to the limiting angular size of PSF that we impose in this work we can use these evaluations.

Several attempts are known to use quantitative parameters from the photometry for automated morphological classification of galaxies. Doi et al. [41] proposed to use comparatively simple and distance free parameters to classify faint distant galaxies – the mean surface brightness SB averaged over the some isophote and concentration index $c_{in}\alpha$ defined with free parameter α. As they pointed out, similar classification may be carried out with the effective surface brightness and concentration index, defined as the ratio of the average surface brightness between different equivalent radii – R_60 and R_80 in their Figure 8. The separation of the galaxies here is worst especially for lower surface brightness - see their figure, cited above.

In Figure 2 the dependence of effective surface brightness from Index of concentration is presented. While by K_means method *the groups of E/S0 galaxies is well outlined* (C ∼5.5) SB-Sd galaxies (C ∼ 2.3) even grouped well by K_means clustering are mixed amongst different surface brightness. As in Doi et al. [41] Figure 8 the separation of galaxies even only as E- and S-ones is poor.

So, an attempt to use the results from surface photometry and concentration index to separate at least early and late type galaxies will be demonstrated now using cluster analysis technique. In Figure 4 the relation of index of concentration C from effective radius $R_{\rm eff}$ and R_{50} is presented. Very *good separation of ES0 an S_ galaxies* is seen. This lead to new possibility for an automatic classification procedure of galaxies. The two morphological groups occupied completely different part of the plot. Straight lines

$$c = a + b * R\_\mathrm{eff} \qquad \text{and} \qquad c = a + b * R\_\mathrm{eff}$$

are the outliers for such procedure. The oulier lines are empirically determinated from the clustering of the parameters – see Figure 4.

Below a new attempt for morphological classification is presented, based on the surface photometry parameters defined. A results of cluster analysis of distribution of effective radius R_eff against Concentration index C is studied for all 1851 objects and for the groups of HSBG and LSBG. Here 5 and 7 clusters are constructed to make selection clearly. For all galaxies such a method allows selection only of *ES0 galaxies – ca. 230 objects*, members of cluster 4 or 7, respectively. The picture becomes much more clear when the objects are preselected based on the surface brightness. Well defined *group of S-galaxies amongst HSBG is selected* and *ca. 80 ES0 galaxies* as well. In the case of LSBG *a com-*

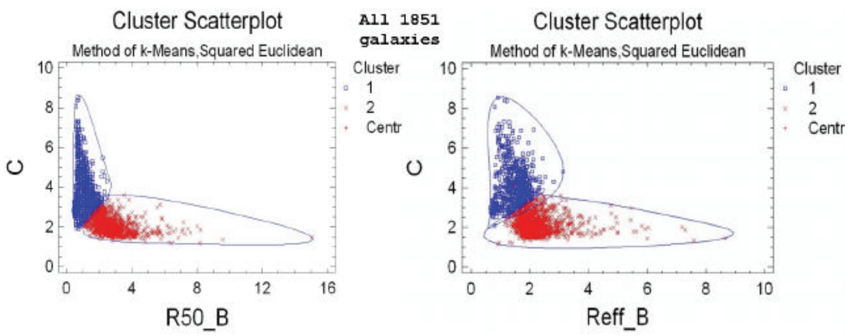

Figure 4. Cluster analysis of: Radius containing 50% of the flux R_{50} and the Effective radius $\mathrm{R}_{\mathrm{eff}}$ against the Index of Concentration distribution.

pact group of ca. 150 ES0 galaxies, members of cluster 2 is oulined and more wide group of S-galaxies. As in Doi et al. [41] intersection of these groups include ca. 400 galaxies impossible to separate in this manner.

Distribution of Position Angles of Galaxies: Histogram of the determined position angles are analysed. Excluding $\pm 90°$ PA, all direction of the orientation are almost equal populated – i.e. *no preferable orientation of the galaxies in the direction of Hercules void is defined*. The only exception is a group of galaxies with large diameters. They tend to define a group with more compact position angles. This could be selection effect as for larger galaxies the accuracy of the position angles determination is better. Nevertheless, radial velocities for these "large" galaxies will clear the question.

Edge-on Galaxies: Some additional information is included in the data. Distribution of axis ratio could codes *edge_on* galaxies - these with *very small axis ratio*. Checking different distribution to fit the observational data we can conclude the objects with Axis_Ratio < 0.4 are a good candidates for edge_on galaxies. It is worth noting that our selection criteria produce a bias against edge-on galaxies. Since we separate objects with the total area within the limiting isophote greater than some minimal threshold, the lower limit for their major-axis diameter of 5 arcsec is valid only for round, face-on galaxies. Thus, for the most elongated galaxies (b/a ~0.1) we will detect all objects with major-axis diameter larger than 50 arcsec. In the same time as one can see at Figure 2, galaxies with different magnitudes/diameters tends to be grouped acc. to axis ratio.

Selecting of Probably Active Galactic Nuclei: Potencial AGN could be hidden amongst the objects with *small effective radius* (R_{eff}, R_{50}) and *comparatively large diameters* $Large\_D$. Cluster membership and the centroids are determined for 5 and 7 clusters (the latter is for better resolution) for all 1851

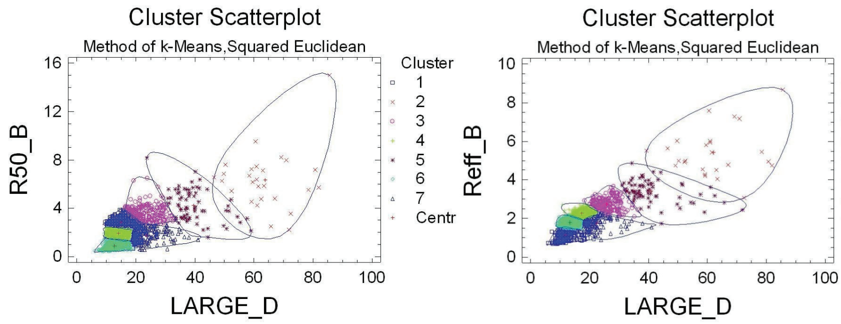

Figure 5. Cluster analysis – looking for probably AGN. The members of cluster 7 are potentialy candidates for AGNs.

galaxies. Both distribution separate well a group of probably AGN-centroids are (1.75, 22.37) and (1.75, 23.50) accordingly, cluster 7 members. Using this method *between 100 and 200 galaxies could be marked as probably AGN*. These distribution for 7 clusters are demonstrated in Figure 5.

4 Conclusions

Petrov et al. [7]presented all the parameters from cluster analysis of an area of $\sim$1 sq.degree in the direction of the Hercules void with automatic object detection software. Ca. 1850 faint galaxies in the wide range of magnitudes $13^m \leq$B$\leq 21^m$ and effective surface brightness 16$\leq \mu_{\rm eff}(\rm B) \leq$24 mag arcsec^{-2} are detected in a field centered at 1600+18 (1950) (Hercules void). Here we demonstrate the possibility of k_Means method of Cluster analysis technique for semiautomatic separating or classifying of galaxies as spiral and elliptical ones and to select Active galactic nuclei among faint galaxies. As a result:

1. Astrometric and photometric parameters for 1851 galaxies are determined.

2. Big excess of bright galaxies (B$<$17.0^m) compared to the galaxy counts-magnitude relation expected in a homogeneous universe assuming Euclidean geometry for three-dimensional space is found. Possibly explanation - the distribution comes at least from two independent selection. Brighter galaxies could be nearer to us and faint end of the distribution will come from the direction of the void itself.

3. Using cluster analysis method for morphological classification of galaxies the groups of E/S0 galaxies (C $\sim$5.5) and SB-Sd galaxies (C $\sim$ 2.3)are

well outlined. Such selection is much better defined for the groups of HSBG and LSBG.

4. A well populated group of ca. 80 galaxies with magnitudes ca. 17.5 and diameters ca. 32–36 arcsec, i.e. faint galaxies with large diameters forms HSBG group.

5. The method outlines a group of ca.30 galaxies probably falling into the category of the so-called Giant LSB galaxies.

6. Cluster analysis results allow us to select ca.20 *edge_on* galaxies.

7. Using cluster analysis AGN candidates are selected - base on R_{eff} and R_50 distribition, ca.120 and 200 AGN candidates respectively are selected amongst the galaxies studied here.

8. No preferable orientation of the galaxies in the direction of Hercules void is defined. Galaxies with D>40 arcsec formed an isolated group.

9. Galaxies with different magnitudes/diameters tends to be grouped acc. to axis ratio.

Acknowledgments

This research partially has made use of the NASA/IPAC Extragalactic Database (NED) which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

References

[1] G. Chincarini and H.J. Rood (1980) *Sky and Telescope*, 364.
[2] S.A. Gregory. and L.A. Thompson, L.A. (1982) *Sci. Am.* **246** 106.
[3] J. Oort (1983) *Ann. Rev. A&Ap.* **21** 373.
[4] H.J. Rood (1988) *Ann. Rev. A&Ap.* **26** 631.
[5] H. Shapley (1934) *MNRAS* **94** 53.
[6] G. Petrov, A. Kniazev, and J. Fried (2005) *AeroSpaceRes* bf 20 120.
[7] G. Petrov, J. Fried, and A. Kniazev (2006) COST A283 MC meeting/Workshop, April 26-30, 2005, Sofia. *Virtual Observatory: Plate Content Digitization, Archive Mining and Image Sequence Processing* Heron Press, Sofia, p.285.
[8] F. Murtagh and A. Heck (1987) *"Multivariate Data Analysis"*, Kluwer Academic Publishers, Dordrecht.
[9] F. Murtagh (2002) *Multivariate data analysis*, WEB course.
[10] E. Feigelson and J. Babu (2003) *"Statistical Challenges in Astronomy"*, Springer Verlag.

[11] G. Fung (2001) *A Comprehensive Overview of Basic Clustering Algorithms*.

[12] P. Franti (2004) *Algorithms in Cluster Analysis*, WEB course.

[13] C.A. Christian, M. Adams, J. Barnes, et al. (1985) *PASP* **97** 363.

[14] A.Y. Kniazev, E.K. Grebel, S.A. Pustilnik, A.G. Pramskij, T.F. Kniazeva, F. Prada, and D. Harbeck (2003) *AJ* **127** 704.

[15] I. Strateva, Z. Ivezic, G.R. Knapp, et al. (2001) *AJ* **122** 1861.

[16] M. Moles, A. del Olmo, and J. Perea (1985) *MNRAS* **213** 365.

[17] G. Chincarini, H.J. Rood, and L. Thompson (1981) *ApJ* **249** L47.

[18] M.A. Arakelian (1975) *Contrib. Bjurak. Obs.* **47** 3.

[19] E. Holmberg (1954) *Medd. Lund. Astron. Obs.* ser.II 136.

[20] K.C. Freeman (1970) *ApJ* **160** 811.

[21] C.D. Impey and G.D. Bothun (1997) *ArAA* **35** 267.

[22] C.D. Impey, G.D. Bothun, and D. Malin (1988) *ApJ* **330** 634.

[23] C.D. Impey, V Burkholder, and D. Sprayberry (2001) *AJ* **122** 2341.

[24] C.D. Impey, D. Sprayberry, M.J. Irwin, and G.D. Bothun (1996) *ApJSuppl* **105** 209.

[25] J.J. Dalcanton (1998) *ApJ* **495** 251.

[26] N. Trentham and R.B. Tully (2002) *MNRAS* **335** 712.

[27] N. Cross and S. Driver (2002) *MNRAS* **329** 579.

[28] J. Ostriker (1993) *ArAA* **31** 589.

[29] P.J.E. Peebles (2001) *ApJ* **557** 495.

[30] L. van Zee, M.P. Haynes, J.J. Salzer, and A.H. Broeils (1997) *AJ* **113** 1618.

[31] A.M.N. Ferguson, R.F.G. Wyse, J.S. Gallagher, and D.A. Hunter (1998) *ApJ* **506** L19.

[32] M. Noguchi (2001) *ApJ* **555** 289.

[33] J.J. Dalcanton, D.N. Spergel, J.E. Gunn, M Smith, and D.P. Schneider (1997) *AJ* **114** 635.

[34] G.D. Bothun, C.D. Impey, D. Malin, and J. Mould (1987) *AJ* **94** 23.

[35] D. Sprayberry, C.D. Impey, G.D. Bothun, and M.J. Irwin (1995) *AJ* **109** 558.

[36] G.D. Bothun, C.D. Impey, and S. McGaugh (1997) *PASP* **109** 745.

[37] S.M. Kent (1985) *ApJSuppl* **59** ??.

[38] K. Kodaira, M. Watanabe, and S. Okamura (1986) *ApJSuppl* **62** 703.

[39] Fioc and Rocca-Volmerange (1999) *A&Ap* **351** 869.

[40] K. Shimasaku, M. Fukugita, M. Doi, et al. (2001) *AJ* **122** 1238.

[41] M. Doi, M. Fukugita, and S. Okamura (1993) *MNRAS* **264** 832.